M.SC. MATHEMATICS

SEMESTER I

SMAM11

Name of the Course : Algebraic Structures

1.a) Let H and K be subgroups of a group G. Then HK is a subgroup of G iff HK=KH.

(OR)

b) State and prove Cauchy's theorem.

2.a) (i) Let G a group. Then G is solvable iff $G^{(m)} = (e)$ for some positive integer.

(ii) Let G be a finite abelian group. Then G is isomorphic to the direct product of its Sylow subgroups.

(OR)

b) If $T \in A(V)$ is nilpotent, of index of nilpotent n_1, then a basis of V can be found such that the matrix of T in this basis has the form

$$\begin{pmatrix} M_{n_1} & 0 & \cdots & 0 \\ 0 & M_{n_2} \cdots & & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & & M_{n_r} \end{pmatrix}$$

Where $n_1 \geq n_2 \geq \ldots \geq n_r$, and where $n_1 + n_2 + \ldots + n_r = \dim_F V$

SMAM12

Name of the Course : Real Analysis – I

1.a) Let f be of bounded variation on [a, b]. If $x \in (a, b]$, let $V(x) = V_f(a, x)$ and put $V(a)=0$. Then every point of continuity of f is also a point of continuity of V. The converse is also true.

(OR)

b) Let $\sum a_n$ be an absolutely convergent series having sum S. Then every rearrangement of $\sum a_n$ also converges absolutely and has sum S.

2.a) State and prove Bernstein Theorem.

(OR)

b) Let $\{f_n\}$ be a sequence of functions defined on a set S. There exists a function 'f' such that $f_n \rightarrow f$ uniformly on S if and only if the following condition (called the Cauchy Condition) is satisfied: For every $\varepsilon > 0$ there exists an N such that $m>N$ & $n>N \Rightarrow |f_m(x) - f_n(x)| < \varepsilon \ \forall \ x \in S$.

SMAM13

Name of the Course : Ordinary Differential Equations

1.a) Find all solutions of $y'' + y = 2\, sinx\, sin2x$

(OR)

b) Find all solutions of the equation

$$x^2 y''' + 2x^2 y'' - x y' + y = 0 \text{ for } x > 0$$

1.a) Find the solutions of

(i) $y' = \frac{x^2 + xy + y^2}{x^2}$

(ii) $y' = \frac{x + y + 1}{2x+2y-1}$

(OR)

b) Determine whether the equation

$$\cos x \cos y\, dx - 2 \sin x \sin y\, dy = 0$$

is exact and solve them.

SMAE11

Name of the Course : Graph Theory and Applications

1.a) Prove that a graph is bipartite if and only if it contains no odd cycles.

(OR)

b) Prove that for any loop-less connected graph; $\kappa(G) \leq \lambda(G) \leq \delta(G)$.

2.a) Prove that every tree has a center consisting of either a single vertex or two adjacent vertices.

(OR)

b) State and prove Cayley's theorem.

SMAE12

Name of the Course : Fuzzy Sets and their Applications

1.a) We consider $\underset{\sim}{A}$ = { (x_1 | 0.2), (x_2 | 0.7),(x_3 |1), (x_4 |0), (x_5 |0.5) }.

and $\underset{\sim}{\boldsymbol{B}}$ = { (x_1 | 0.5), (x_2 | 0.3),(x_3 |1), (x_4 |0.1), (x_5 |0.5) }.

find $\overline{\underset{\sim}{A}}$, $\overline{\underset{\sim}{B}}$, $\underset{\sim}{A} \cap \overline{\underset{\sim}{B}}$, $\overline{\underset{\sim}{A}} \cap \underset{\sim}{B}$ & $\underset{\sim}{A} \oplus \underset{\sim}{B}$

(OR)

b) Let $E_1 = E_2 = R^+$, where R^+ is the set of nonnegative real numbers. Let $x \in R^+$ and y$\in R^+$

and consider the product set $R^+ \times R^+$. Then the relation $y > x$ defines a fuzzy graph in ${R^+}^2$.

2.a) Give an example for disjunctive sum of two relations:

(OR)

b) Consider two fuzzy relations $x\underset{\sim}{R_1}y$ and $y\underset{\sim}{R_2}z$, $where$ x, y and $z \in R^+$. We suppose

$\mu_{\underset{\sim}{R_1}}(x, y) = e^{-k(x-y)^2}, k \geq 1$ & $\mu_{\underset{\sim}{R_2}}(y, z) = e^{-k(y-z)^2}, k \geq 1$. Determine $\mu_{\underset{\sim}{R_1} \circ \underset{\sim}{R_2}}(x, z)$.

SEMESTER II

SMAM21

Name of the Course : Advanced Algebra

1.a) The element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

(or)

b) A polynomial of degree n over a field can have at most n roots in any extension field.

2. a) Let K be the field of complex numbers and let F be the field of real numbers.
Compute $G(K, F)$.

(or)

b) Let F be a finite field; then F has p^m elements where the prime number p is the characteristic of F.

SMAM22

Name of the Course : Real Analysis – II

1.a) The following statements regarding the set E are equivalent:

(i) E is measurable

(ii) $\forall \varepsilon > 0,70$, an open set, $0 \supseteq E$ such that $m^*(O - E) \leqslant \varepsilon$

(iii) $\exists G, aG_\delta -$ set, $G \supseteq E$ such that $m^*(G - E) = 0,$

(ii)* $\forall \varepsilon > 0,7$ F, a closed set, $F \subseteq E$ such that $m^*(E - E) \leq \varepsilon$

(iii) * $\exists F$, an $F_\sigma -$ set, $F \subseteq E$ such that $m^*(E - F) = 0$

(OR)

b) Let c be any real number and let $f \neq g$ be real-valued measurable functions defined on the some measurable set E. Then $f + c, cf, f + g, f - g$ and fg are also measurable.

2.a) Let f of g be non-negative measurable functions

(i) If $f \leqslant g$, then $\int f dx \leqslant \int g dx$

(ii) If A is a measurable set and $f \leqslant g$ on A, then $\int_A f dx \leqslant \int_A g dx$

(iii) If $a \geqslant 0$, then $\int a f dx = a \int f dx$

(iv) If A & B are measurable sets and $A \subset B$, then $\int_A f dx \geqslant \int_B f dx$.

(OR)

b) If g is of bounded variation on $[0, \delta]$. Then $\lim_{\alpha \to +\infty} \frac{2}{\pi} \int_0^\delta g(t) \frac{\sin \alpha t}{t} dt = g(0+)$

SMAM23

Name of the Course : Partial Differential Equations

1.a) Determine the region in which the equation. $y^2 u_{xx} - x^2 U_{yy} = 0$ is hyperbolic, parabolic, elliptic, and trans form the equation in the respective region to the canonical form.

(OR)

b) Determine the solution of $u_{tt} = c^2 u_{xx}$ $0 < x < l,\ t > 0$.

$$
\begin{aligned}
&u(x,0) = \sin\left(\frac{\pi x}{l}\right). && 0 \le x \le l\\
&u_t(x,0) = 0. && 0 \le x \le l\\
&u(0,t) = 0. && t \ge 0\\
&u(l,t) = 0. && t \ge 0.
\end{aligned}
$$

2.a) State and prove mean value theorem.

(OR)

b) State and prove uniqueness theorem.

SMAE21

Name of the course : Mathematical Statistics

1.a) For each of the following, find the constant c so that $f(x)$ satisfies

the conditions of being a p.d.f. of one random variable X.

(i) $f(x) = cx, x = 1, 2, 3, \ldots$, zero elsewhere.

(ii) $f(x) = cx, 0 < x < (X)$, zero elsewhere.

(OR)

b) Let $f(x) = x/15, x = 1,2,3,4,5$, zero elsewhere, be the p.d.f. of X.

Find $Pr\ (X = 1\ or\ 2)$, $Pr\ (1/2 < X < 5/2)$, and $Pr\ (1\ X\ 2)$.

2. a) Let $u(X)$ be a nonnegative function of the random variable X. If $E[u(X)]$ exists, then, for every positive constant c.

(OR)

b) State and prove Chebyshev's Inequality.

SMAE22

Name of the Course : Operations Research

1.a) Using Vogel approximation method find the basic solution to the following transportation method.

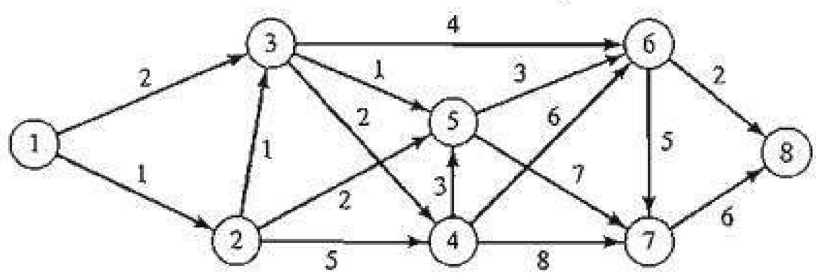
	W_1	W_2	W_3	Supply
F_1	8	10	12	900
F_2	12	13	12	1000
F_3	14	10	11	1200
Demand	1200	1000	900	

(OR)

b) The network in Figure gives the distances in miles between pairs of cities 1,2, ., . , and 8. Use Dijkstra's algorithm to find the shortest route between the following cities:

a. Cities 1 and 8

b. Cities 1 and 6



2.a) The demand rate of a particular item is 12000 units per year. The set-up cost per unit is Rs. 350 and the holding cost is Rs.20 per unit per month. If no shortages are allowed and the replacement is instantaneous, determine (i) the optimum lot size, (ii)the optimum scheduling period, and

(iii)minimum total expected annual cost.

(OR)

b) A repairman is to be hired to repair machines which break down at an average rate of 3 per hour. The breakdown follows a Poisson distribution. Non-productive time of a machine is considered to cost Rs.10 per hour. Two repairmen have been interviewed of whom one is slow but charges less and the other is fast but more expensive. The slow repairman charges Rs.5 per hour and services breakdown machines at the rate of 4 per hour. The fast repairman demands 7 Rs per hour, but services breakdown machines at an average rate of 6 per hour. Which repairman should be hired?

SMAS21

Name of the course : Mathematical Documentation using LaTeX

1.a) Explain about the basis of a latex file.

(OR)

b) Explain about table of contents.

2.a) Explain about generalized lists.

(OR)

b) Explain about footnotes and marginal notes.

SEMESTER III

SMAM31

Name of the Course - Complex Analysis

1.a) State and prove Lucas theorem

(OR)

b) State and prove Abel's theorem

2.a) compute $\int_{|z|=2} \frac{dz}{z^2+1}$

(OR)

b) Suppose that $\phi(\zeta)$ is continuous an arc ν. Then the function $F_n(z) = \int_\gamma \frac{\phi(\zeta)d\zeta}{(\zeta-2)^n}$ is analytic $F_n(z)$. In each of the region determined by ν and the its derivatives is $F_n^{'}(z) = nF_{n+1}(z)$.

SMAM32

Name of the Course - Probability Theory

1.a) Let $\{A_n\}, n = 1,2, ...,$ be a non -increasing of events and let A be their product. Then $P(A) = \lim_{n\to\infty} P(A_n)$.

(OR)

b) The expected value of the product of an arbitrary finite number of independent random variables \, whose expected values exist, equals the product of the expected values of these variables.

2.a) Find the density function of the random variable X, whose characteristic function is

$$\phi_{1(t)} = \begin{cases} 1-|t| & for\ |t| \le 1 \\ 0 & for\ |t| > 1 \end{cases}$$

(OR)

b) Let $(X_1,\ X_2, ..., X_n\ )$ be an n-dimensional random variable with a normal distribution and let $Y_1,\ Y_2, ..., Y_n$, where $r \le n$, be linear fractions of random variable $X_j (j = 1,2, ..., n)$. Then the

random variable $(Y_1,\ Y_2, ..., Y_n)$ also has a normal distribution.

SMAM33

Name of the Course : Topology

1.a) (i) Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that each open set U of X and each $x \in U$, there is an element C of $\mathcal{C}$ such that $x \in C \subset U$. Then $\mathcal{C}$ is a basis for the topology of X.

(ii) Let Y be a subspace of X. Then a set A is closed in Y iff A equals the intersection of a closed set of X with Y.

(OR)

b) Let $f: A \to \prod X_\alpha$ be given by the equation $f(a) = \left(f_\alpha(a)\right)_{\alpha \in J}$, where $f_\alpha: A \to X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology. Then the function f is continuous iff each function f_α is continuous.

2.a) If Y is a subspace of X, a separation of Y is a pair of disjoint non-empty sets A and B whose union is Y, neither of which contains a limit point of the other. The space Y is connected if there exist no separation of Y.

(OR)

b) Prove the followings:

(i) A subspace of a first countable space is first countable.

(ii) A countable product of first countable space is first countable.

(iii) A subspace of a second countable space is second countable.

(iv) A countable product of second countable spaces is second countable.

SMAM34

Name of the Course : Calculus of Variations and Integral Equations

1.a) Determine the point on the curve of intersection of the surface z = xy + 5, x + y + z = 1 which is nearest to the origin.

(OR)

b) Derive Transversality condition

2.a) Derive Green's Function.

(OR)

b) Solve the integral equation y(x) = $\lambda \int (1 - 3x\xi)(\xi) d\xi + F(x)$, for non-homogenous case.

SMAE31

Name of the Course : Mechanics

1.a) State and prove principle virtual work

(OR)

b) A particle of mass m is suspended by a massless wire of length $r = a + b\cos\omega t$, $a, b > 0$. To form a spherical pendulum, find the equation of motion.

2.a) Prove that Bilinear covariant is invariant with respect to canonical transformation

(OR)

b) Derive Hamilton principle function.

SMAS31

Name of the Course : Programming in C++

1. a) Write the differences between structures and unions

(OR)

b) Write a program which illustrates Function Overloading.

2. a) Write a program which illustrates the use of object arrays.

(OR)

b) Write some Operator overloading examples.